

On the sound field from a moving source in a viscous medium

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Based on a one-dimensional model, a wave-theoretic analysis of the sound field from a moving, harmonic source in a viscous medium is developed. When the source is inbound to the receiver, the field consists of an attenuated propagating wave with Döppler up-shifted frequency. A similar wave is present when the source is outbound from the receiver but with down-shifted frequency. Also present on the outbound run is an evanescent wave that appears at the instant the source passes the receiver. The evanescent wave is very highly attenuated and exists only in the presence of both source motion and dissipation. Expressions are formulated for the frequency and attenuation coefficient of the two propagating waves and the evanescent wave. The attenuation of the propagating waves scales with the square of the frequency, which is a characteristic of viscous dissipation; and the attenuation is strongly asymmetrical, being significantly higher on approach than departure. The asymmetry in the attenuation arises partly from the upward and downward Döppler shifts in the frequency on approach and departure, respectively. In addition, the attenuation is skewed by the factor $(1 \pm \beta)^{-1}$, where the lower (upper) sign applies on approach (departure) and β is the Mach number of the source. At a Mach number of 0.85, the ratio of the inbound to outbound attenuation is 2000. © 2003 Acoustical Society of America. [DOI: 10.1121/1.1624068]

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I. INTRODUCTION

Sound from a moving source is usually discussed on the implicit assumption that the medium supporting the acoustic field is nondispersive and nondissipative.^{1–5} In such a situation, the familiar Döppler effect is observed, taking the form of an up-shift or down-shift in received frequency, according to whether the source is inbound to or outbound from the receiver.

The Döppler frequency shift has been exploited recently in a series of underwater acoustics experiments conducted off the coast of La Jolla, southern California^{6,7} in which a propeller-driven light aircraft was used as an acoustic source for measuring the speed of sound in a marine sediment. The acoustic coupling across the air–sea interface and the ocean–sediment interface is sufficiently good for the sound of the aircraft’s propeller, which takes the form of a series of harmonics spanning the frequency range from about 80 Hz to 1 kHz, to be detectable not only on hydrophones in the water column but also on a receiver buried approximately 75 cm deep in the sediment. The technique that has been developed exploits the difference in Döppler-shifted frequencies on aircraft approach and departure. This Döppler difference frequency is inversely proportional to the speed of sound in the local medium where the receiver is located, that is, the sediment in the case of a buried sensor. It has been found that the difference-frequency technique yields the low-frequency sediment sound speed with reasonable precision.^{6,7}

Like the sound speed, the acoustic attenuation in a marine sediment is also of considerable interest, although an

entirely satisfactory technique for extracting it from propeller-noise data has not yet been developed. The challenge of determining the attenuation in the sediment using aircraft noise involves several questions concerning the effects of dissipation on the sound field from a moving source. Before dealing with the complexities of the atmosphere-ocean-sediment environment, however, a much simpler problem is addressed in this article, with a view to establishing some of the fundamental properties of waves from a moving sound source in a dissipative medium.

A simple one-dimensional scenario is considered (Fig. 1) in which a moving, nonaccelerating, harmonic acoustic source and a stationary receiver are located in a homogeneous, isotropic viscous fluid. The source is assumed to follow a rectilinear path passing through the receiver. A full wave-theoretic analysis of the received field is developed, which leads to a solution involving three waveforms. On the inbound run, a propagating Döppler up-shifted wave appears at the receiver showing an attenuation that is greater than

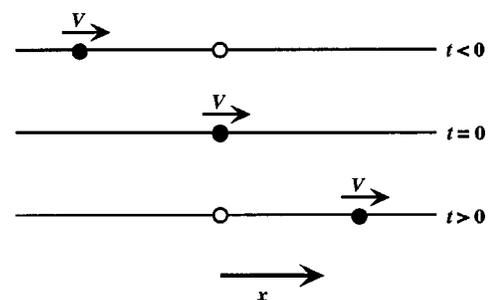


FIG. 1. Geometry of the one-dimensional moving-source (solid circle, speed V) and stationary-receiver (open circle) problem. For negative times the source is inbound to the receiver, they are coincident at time zero, and the source is outbound from the receiver for positive times.

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would be observed in the absence of source motion. Outbound, a propagating Döppler down-shifted wave appears, but with attenuation less than that seen when the source is stationary. The attenuation, of course, scales with the viscosity of the medium, and the asymmetry in attenuation, inbound versus outbound, is a direct result of the motion of the source. In addition to the two propagating waves, an evanescent wave also exists, appearing at the instant the source passes the stationary receiver. The evanescent wave is extremely short-lived and appears only when source motion and viscosity are both present. Expressions are given for the wave speeds and attenuations of all three waves.

II. THE WAVE EQUATION

If ϕ is the velocity potential of the acoustic field, Q is the source strength, and x is the direction of the line connecting the source and receiver, then the wave equation describing the one-dimensional field in the viscous medium⁸ is

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{c_0^2} \frac{\partial^2 \phi}{\partial t^2} + \gamma \frac{\partial^3 \phi}{\partial x^2 \partial t} = -Q \delta(x - Vt) f(t), \quad (1)$$

where c_0 is the speed of sound in the medium in the low-frequency limit, $\delta(\dots)$ is the Dirac delta function, which accounts for the source motion, $f(t)$ is the time series of the source, and $V \geq 0$ is the speed of the source. According to Eq. (1), the source, located at $x = Vt$, is coincident with a receiver at $x = 0$ when $t = 0$. As shown in Fig. 1, the source is inbound to the receiver when $t < 0$ and outbound when $t > 0$. The third term on the left of Eq. (1) represents the effects of viscous dissipation and γ is the dissipation coefficient, which scales with the dynamic viscosity, μ :

$$\gamma = \frac{4\mu}{3\rho c_0^2}, \quad (2)$$

where ρ is the density of the medium.

To solve for the field, ϕ , two Fourier transforms are applied to Eq. (1), one with respect to time, t , and the second to distance, x . The Fourier transform and its inverse are, respectively,

$$\phi_q = \int_{-\infty}^{\infty} \phi e^{-iqz} dz \quad (3a)$$

and

$$\phi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_q e^{iqz} dq, \quad (3b)$$

where $i = \sqrt{-1}$ and the transform variable appearing as a subscript denotes the transformed field, which is a convenient notation when several integral transforms are applied in sequence to a partial differential equation, as is the case here.

On Fourier transforming with respect to x , Eq. (1) reduces to

$$-p^2 \left(1 + \gamma \frac{\partial}{\partial t} \right) \phi_p - \frac{1}{c_0^2} \frac{\partial^2 \phi_p}{\partial t^2} = -Q e^{-ipVt} f(t), \quad (4)$$

where the transform variable p is the wave number. A further Fourier transform, with respect to time, t , yields an algebraic expression for the doubly transformed field:

$$\phi_{p\omega} = -\frac{QF(\omega + pV)}{k^2 - p^2(1 + i\omega\gamma)}, \quad (5)$$

where the transform variable ω is the angular frequency, $F(\omega)$ is the temporal Fourier transform of the source waveform, $f(t)$, and $k = \omega/c_0$ is the acoustic wave number. Note that the effect of source motion is fully accounted for by the shifted frequency in the Fourier transform of the source function appearing in the numerator of Eq. (5).

To perform the inverse Fourier transforms necessary to recover the velocity potential, ϕ , the source function, $f(t)$, must be specified. Assuming an harmonic source of angular frequency Ω , the source function may be expressed as

$$f(t) = f_0 \cos(\Omega t + \psi), \quad (6)$$

where f_0 is the amplitude and ψ is the phase of the wave. Accordingly, the Fourier transform of the source function, $F(\omega)$, is

$$F(\omega) = A \delta(\omega - \Omega) + A^* \delta(\omega + \Omega), \quad (7a)$$

where the asterisk denotes complex conjugation and complex coefficient is

$$A = \pi f_0 e^{i\psi}. \quad (7b)$$

On substituting Eq. (7a) into Eq. (5), the expression for the doubly transformed field takes the form

$$\phi_{p\omega} = -Q \frac{A \delta(\omega - \Omega + pV) + A^* \delta(\omega + \Omega + pV)}{k^2 - p^2(1 + i\omega\gamma)}. \quad (8)$$

The presence of the delta functions in the numerator of Eq. (8) allows the inverse Fourier transform with respect to angular frequency, ω , to be performed immediately:

$$\begin{aligned} \phi_p &= -\frac{Q}{2\pi} \\ &\times \int_{-\infty}^{\infty} \frac{[A \delta(\omega - \Omega + pV) + A^* \delta(\omega + \Omega + pV)]}{k^2 - p^2(1 + i\omega\gamma)} e^{i\omega t} d\omega \\ &= -\frac{Q}{2\pi} \left\{ \frac{A e^{i(\Omega - pV)t}}{(\Omega - pV)^2/c_0^2 - p^2[1 + i(\Omega - pV)]} \right. \\ &\quad \left. + \frac{A^* e^{-i(\Omega + pV)t}}{(\Omega + pV)^2/c_0^2 - p^2[1 - i(\Omega + pV)]} \right\}. \quad (9) \end{aligned}$$

Application of the remaining inverse Fourier transform, with respect to wave number p , yields an integral expression for the velocity potential:

$$\phi = -\frac{Q}{2\pi^2} \operatorname{Re} \left\{ A e^{i\Omega t} \int_{-\infty}^{\infty} \frac{e^{ip(x-Vt)}}{-iV\gamma p^3 - p^2(1 - V^2/c_0^2 + i\Omega\gamma) - 2pV\Omega/c_0^2 + \Omega^2/c_0^2} dp \right\}, \quad (10)$$

where Re denotes the real part of the expression in parenthesis.

The denominator of the integrand in Eq. (10) is a cubic in the wave number p and therefore has three roots. Note that if either V or γ goes to zero, representing, respectively, a motionless source or an inviscid medium, the third-order term vanishes and the denominator reduces to a quadratic. The two roots of the quadratic represent simple poles in the complex p -plane. For the case of a moving source in an inviscid medium, one of these poles yields a Doppler-shifted propagating wave with up-shifted frequency when $t < 0$ and the other a propagating wave with down-shifted frequency when $t > 0$. In the other situation, with a motionless source in a viscous medium, the two poles yield identical propagating waves, one for the case when the source position is positive and the other for when it is negative relative to the receiver.⁸ These waves exhibit frequency dispersion and attenuation, the latter scaling as ω^2 at low frequencies ($\omega\gamma \ll 1$) and $\omega^{1/2}$ at high frequencies ($\omega\gamma \gg 1$). Thus, for source motion alone or dissipation alone the solution in Eq. (10) correctly represents familiar, well-understood behavior. When source motion and dissipation are both present, and only then, the coefficient of the cubic term in the denominator of the integrand in Eq. (10) is nonzero, making the situation a little more complicated.

III. THE CUBIC DENOMINATOR

The presence of the cubic in Eq. (10) suggests that three types of wave must be present in the field detected at the receiver. Two of these wave forms will be the familiar Doppler up-shifted (on approach) and down-shifted (on departure) propagating waves, each of which will be attenuated due to the viscosity of the medium. We may anticipate that the attenuation of the up- and down-shifted waves will be asymmetrical, to a degree that is determined by the speed of the source. The third waveform will turn out to be extremely short-lived in most circumstances, appearing at the instant the source passes the receiver and subsequently decaying rapidly in time and space.

To examine the properties of the three waves, it is necessary to find the roots of the cubic in the wave number p . For this purpose, it is convenient to express the denominator in Eq. (10) in the form

$$D = i\beta L p^3 - p^2(1 - \beta^2 + iqL) - 2q\beta p + q^2, \quad (11)$$

where

$$\beta = \frac{V}{c_0} \quad (12)$$

is the Mach number of the source,

$$q = \frac{\Omega}{c_0} \quad (13)$$

is the wave number of the source (with units of inverse length), and

$$L = c_0 \gamma \quad (14)$$

is a viscous dissipation length. Notice that for most practical situations the dimensionless product qL is very much less than unity, a fact that is used below in deriving approximate roots for the cubic in Eq. (10).

By writing the cubic in the standard form

$$D = ap^3 + 3bp^2 + 3cp + d = 0, \quad (15)$$

the coefficients may be identified as

$$a = i\beta L, \quad (16a)$$

$$b = -(1 - \beta^2 + iqL)/3, \quad (16b)$$

$$c = -2q\beta/3, \quad (16c)$$

and

$$d = q^2. \quad (16d)$$

The three roots of Eq. (15) can be expressed analytically through a standard procedure,⁹ which relies on the substitution $y = (ap + b)$ to eliminate the second-order term from the cubic. With $j = 1, 2$ or 3 , the three roots are of the form

$$p_j = \frac{y_j - b}{a}, \quad (17)$$

where

$$y_j = As_j + Bs_j^2, \quad (18)$$

$$A = \left\{ \frac{-G + \sqrt{G^2 + 4H^3}}{2} \right\}^{1/3}, \quad (19)$$

$$B = \left\{ \frac{-G - \sqrt{G^2 + 4H^3}}{2} \right\}^{1/3}, \quad (20)$$

$$H = ac - b^2, \quad (21)$$

and

$$G = a^2d - 3abc + 2b^3. \quad (22)$$

The s_j in Eq. (18) are the cube roots of unity:

$$s_1 = s_1^2 = 1, \quad (23a)$$

$$s_2 = s_3^2 = e^{2i\pi/3} = \frac{-1 + i\sqrt{3}}{2}, \quad (23b)$$

and

$$s_3 = s_2^2 = e^{4i\pi/3} = \frac{-1 - i\sqrt{3}}{2}. \quad (23c)$$

Although Eqs. (17)–(23) represent the formal solution for the roots of a cubic, the formulation is too cumbersome to provide much insight into the physics of the sound field

from a moving source in a viscous medium. However, a more revealing, approximate form of this solution may be obtained by recognizing that, for subsonic source speeds and realistic levels of viscosity, the magnitude of the coefficient of the third-order term in Eqs. (11) and (15) is small compared with unity: $|a| = \beta L \ll 1$. It is appropriate, therefore, to expand the solutions for the three roots in Eq. (17) to first-order in the small coefficient, a . (*N.B.* This requires that G^2 and H^3 be expanded to third-order in a .) To this level of approximation, the three roots are found to be

$$p_1 = -\frac{3b}{a} + \frac{c}{b} - \frac{a}{9b^3}(bd - 3c^2), \quad (24)$$

$$p_2 = -\frac{1}{2b} \left[c - i \sqrt{\frac{4bd - 3c^2}{3}} \right] + \frac{a}{18b^3} \left[(bd - 3c^2) + \frac{i3\sqrt{3}c(bd - c^2)}{\sqrt{4bd - 3c^2}} \right], \quad (25)$$

and

$$p_3 = -\frac{1}{2b} \left[c + i \sqrt{\frac{4bd - 3c^2}{3}} \right] + \frac{a}{18b^3} \left[(bd - 3c^2) - \frac{i3\sqrt{3}c(bd - c^2)}{\sqrt{4bd - 3c^2}} \right]. \quad (26)$$

The latter two expressions, for p_2 and p_3 , are Taylor series in a , whereas Eq. (24) for p_1 is a Laurent series, possessing a term in a^{-1} . When the expressions for the coefficients in Eqs. (16) are substituted into Eqs. (24)–(26), a little algebra yields the following:

$$p_1 = \frac{N^2 - \beta^2}{i\beta L} + \frac{2q\beta}{N^2 - \beta^2} - \frac{i\beta L q^2 (N^2 + 3\beta^2)}{(N^2 - \beta^2)^3}, \quad (27)$$

$$p_2 = \frac{q}{N + \beta} \left[1 + \frac{i\beta q L}{2N(N + \beta)^2} \right], \quad (28)$$

and

$$p_3 = -\frac{q}{N - \beta} \left[1 - \frac{i\beta q L}{2N(N - \beta)^2} \right], \quad (29)$$

where

$$N = \sqrt{1 + iqL}. \quad (30)$$

Note that the expression for p_1 in Eq. (27) goes to $-i\infty$ when either the viscosity reduces to zero or the source is stationary. In general, p_1 possesses a large imaginary component, indicating that this pole represents an evanescent wave. The remaining two poles, p_2 and p_3 , which also possess imaginary components but with relatively small magnitudes, represent attenuated propagating waves with down-shifted ($t > 0$) and up-shifted ($t < 0$) Doppler frequencies, respectively.

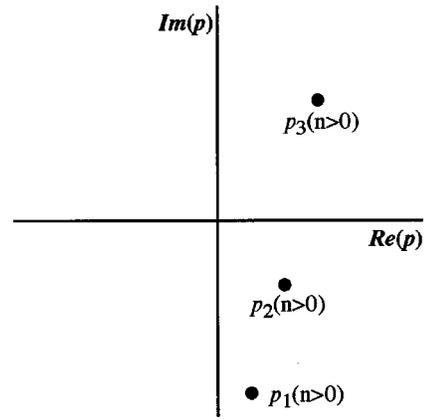


FIG. 2. The complex p -plane showing the positions of the three poles of the integrand in Eq. (10).

IV. THE VELOCITY POTENTIAL

To evaluate the Fourier inversion integral in Eq. (10), it is necessary to identify the positions of the poles in the complex p -plane. From Eqs. (27)–(29), poles p_1 and p_2 lie in the fourth quadrant, while p_3 is in the first quadrant, as shown in Fig. 2. Now the field expression in Eq. (10) can be expressed in the form

$$\phi(t) = \frac{Q}{2\pi^2\beta L} \times \text{Re} \left\{ iA e^{i\Omega t} \int_{-\infty}^{\infty} \frac{e^{-ipVt}}{(p-p_1)(p-p_2)(p-p_3)} dp \right\}, \quad (31)$$

where x , representing the receiver position, has been set to zero. For negative times, when the source is inbound to the receiver, a D-contour of infinite radius around the top half-plane, enclosing p_3 , yields the solution

$$\phi(t < 0) = -\frac{Q}{\pi\beta L} \text{Re} \left\{ A \frac{e^{i(\Omega t - p_3 V t)}}{(p_3 - p_1)(p_3 - p_2)} \right\}. \quad (32)$$

On the outbound track, when $t > 0$, the D-contour around the lower half-plane must be taken, enclosing p_1 and p_2 , in which case the expression for the field is

$$\phi(t > 0) = \frac{Q}{\pi\beta L} \text{Re} \left\{ A \frac{e^{i(\Omega t - p_2 V t)}}{(p_2 - p_1)(p_2 - p_3)} + A \frac{e^{i(\Omega t - p_1 V t)}}{(p_1 - p_2)(p_1 - p_3)} \right\}. \quad (33)$$

Since $qL \ll 1$ for realistic values of the viscosity, only the leading-order terms need be retained in the factors outside the exponential functions in Eqs. (32) and (33), an approximation which yields

$$\phi(t < 0) = -\frac{Q}{2\pi q} \text{Re} \{ iA e^{i(\Omega t - p_3 V t)} \} \quad (34)$$

and

$$\phi(t>0) = -\frac{Q}{2\pi q} \operatorname{Re}\{iAe^{i(\Omega t - p_2 V t)}\} - \frac{Q\beta L}{\pi(1-\beta^2)^2} \operatorname{Re}\{Ae^{i(\Omega t - p_1 V t)}\}. \quad (35)$$

The asymmetry between the inbound [Eq. (34)] and outbound [Eq. (35)] wave fields is apparent from the presence of the second term on the right of Eq. (35). On approach, the received field consists of a propagating wave with Doppler up-shifted frequency and on departure the corresponding down-shifted propagating wave is present; but at the instant the source passes the receiver an evanescent wave also appears, associated with the pole p_1 .

It is worth mentioning that, if the time-derivative of the velocity potential is taken, to yield the pressure field, the amplitude of the propagating wave to leading-order in qL is found to scale as $(1 \pm \beta)^{-1}$, where the minus and plus signs apply to the inbound and outbound fields, respectively. Thus, the source motion raises (lowers) the amplitude of the inbound (outbound) pressure field, a result that is familiar from the elementary theory of sound from a moving source in a nondispersive medium.¹⁰

V. THE PROPAGATING WAVE

In a viscous medium, the inbound and outbound propagating waves from the moving source exhibit Doppler-shifted frequencies and also attenuation. The shifted frequencies are obtained from the real part of the coefficient of t in the arguments of the exponential functions for the propagating waves [Eqs. (34) and (35)]. The attenuation coefficient is the magnitude of the imaginary part of the wave numbers p_2 (outbound) and p_3 (inbound) in Eqs. (28) and (29), respectively.

To first order in qL , the expressions for the poles p_2 and p_3 are

$$p_{2,3} \approx \pm q \left[\frac{1}{(1 \pm \beta)} - \frac{iqL}{2(1 \pm \beta)^3} \right], \quad (36)$$

where the upper sign yields p_2 (outbound) and the lower sign p_3 (inbound). From the real part of Eq. (36), the received (angular) frequencies on approach (minus sign) and departure (plus sign), to first-order in qL , are easily shown to be

$$\Omega_{\text{in}} \approx \frac{\Omega}{(1-\beta)} \quad (37a)$$

and

$$\Omega_{\text{out}} \approx \frac{\Omega}{(1+\beta)}. \quad (37b)$$

Thus, to this order of approximation, the inbound and outbound frequency shifts due to source motion are the same as would be observed in an inviscid medium.

Turning to the imaginary part of Eq. (36), the attenuation coefficient of the pressure field when the source is inbound to the receiver is

$$\alpha_{\text{in}} \approx \frac{q^2 L}{2(1-\beta)^2} \frac{1}{(1-\beta)}, \quad (38a)$$

and, when the source is outbound,

$$\alpha_{\text{out}} \approx \frac{q^2 L}{2(1+\beta)^2} \frac{1}{(1+\beta)}. \quad (38b)$$

According to these expressions, the source motion affects the amplitudes of the propagating waves arriving at the receiver by introducing an asymmetrical modification to the viscous attenuation. It is evident that, even for modest source speeds, the attenuation may be significantly higher on approach than departure.

Part of the asymmetry may be attributed to the square of the frequency, $[\Omega/(1 \pm \beta)]^2$, which appears as a factor in Eqs. (38). Such a square-law frequency scaling is a familiar characteristic of viscous dissipation. From the up- and down-shifts in frequency, the attenuation is enhanced on approach and reduced on departure. The asymmetrical effect of the Doppler-shifted frequency on the inbound and outbound attenuations is reinforced by a further asymmetry, represented by the factors $1/(1 \mp \beta)$ appearing in Eqs. (38). [*N.B.* These extra factors derive directly from the third-order term in the cubic in Eq. (11), as may be appreciated by neglecting this term and solving the resulting quadratic for $p_{2,3}$. Identical results to those in Eqs. (38) are obtained, except that the factors $1/(1 \mp \beta)$ are absent.] The full asymmetry between the attenuation on approach and departure is evident from the ratio

$$\frac{\alpha_{\text{in}}}{\alpha_{\text{out}}} = \frac{(1+\beta)^3}{(1-\beta)^3} = 2.42, \quad (39)$$

where the numerical value corresponds to a Mach number $\beta=0.147$, representative of a light-aircraft source traveling at $V=50$ m/s in an atmosphere with sound speed $c_0=340$ m/s. Figure 3 shows the expression in Eq. (39) for the ratio of inbound and outbound attenuations plotted as a function of the Mach number, β . Note that at a Mach number of 0.85, representative of the cruise speed of a commercial jet transport, the attenuation coefficient as the aircraft approaches an observer is a factor of 2000 greater than that as it departs. This asymmetry in the level of the perceived sound is, of course, in addition to any that may arise from the aspect dependence of the source itself, which in the case of a jet or indeed a propeller aircraft may be quite pronounced.

The absolute values of the inbound and outbound attenuation coefficients depend on the dynamic viscosity and the density of the medium. For air¹¹ at 20 °C, $\mu=1.8 \times 10^{-5}$ kg/m s and $\rho=1.21$ kg/m³. At a frequency of 1 kHz and with $\beta=0.147$, Eqs. (38) then yield $\alpha_{\text{in}}=1.6 \times 10^{-5}$ Nepers/m (1.4×10^{-4} dB/m) and $\alpha_{\text{out}}=6.6 \times 10^{-6}$ Nepers/m (5.8×10^{-5} dB/m). In fact, these values are likely to be underestimates of the inbound and outbound attenuation, since thermal conduction losses give rise to an “effective” viscosity, which is approximately 50% higher than the dynamic viscosity.^{12,13} If, in addition, molecular relaxation effects are included in the effective viscosity, its value increases by about one order of magnitude, raising the

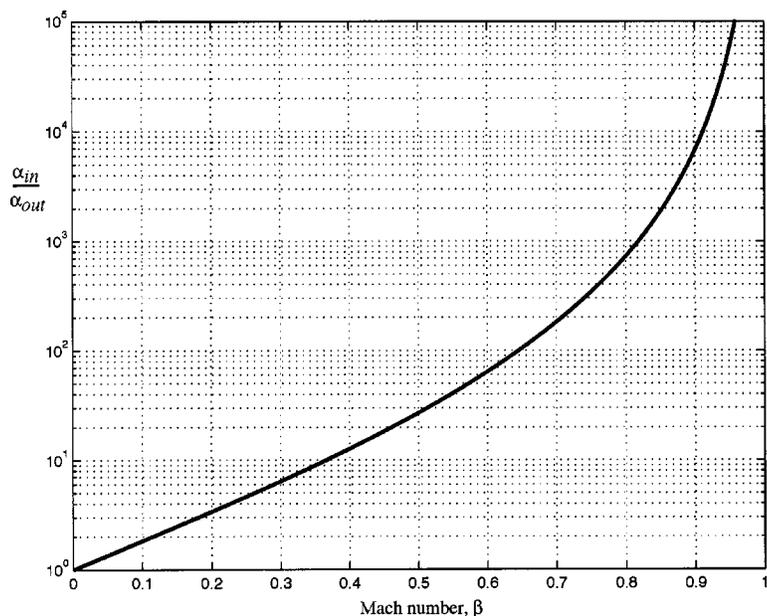


FIG. 3. Ratio of the attenuation of the inbound and outbound propagating waves as a function of Mach number [Eq. (38)].

attenuations at 1 kHz to $\alpha_{in} \approx 2.4 \times 10^{-4}$ Nepers/m (2.1×10^{-3} dB/m) and $\alpha_{out} \approx 9.9 \times 10^{-5}$ Nepers/m (8.7×10^{-4} dB/m).

VI. THE EVANESCENT WAVE

The second term on the right of Eq. (35) represents an evanescent wave that appears when the source passes the receiver and which persists very briefly after time $t=0$. Under typical atmospheric conditions, the decay of this wave is extremely rapid, the attenuation coefficient being given by the magnitude of the imaginary part of the wave number p_1 . From Eq. (27), the Laurent series for p_1 , to first-order in qL , is

$$p_1 \approx -i \frac{(1-\beta^2)}{\beta L} + q \frac{(1+\beta^2)}{\beta(1-\beta^2)} - i \frac{\beta q^2 L (3+\beta^2)}{(1-\beta^2)^3}. \quad (40)$$

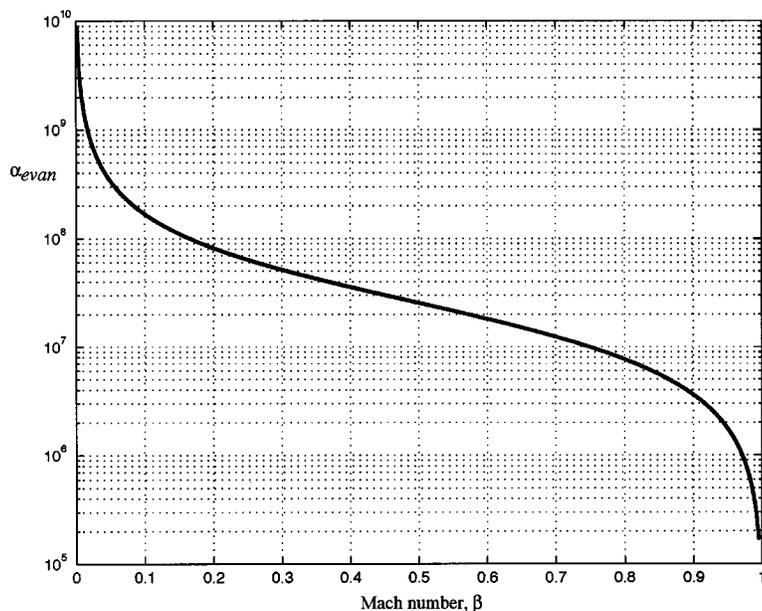


FIG. 4. Attenuation (nepers/m) of the evanescent wave as a function of Mach number [Eq. (41)], with $L = 5.9 \times 10^{-8}$ m, appropriate to air.

For subsonic source speeds ($\beta < 1$), only the leading-order term of the attenuation coefficient is significant:

$$\alpha_{evan} \approx \frac{(1-\beta^2)}{\beta L}, \quad (41)$$

which is independent of the source frequency, Ω .

Unlike the propagating waves, where the attenuation is proportional to the dissipation constant, γ , the attenuation of the evanescent wave, α_{evan} scales as the reciprocal of γ . Since α_{evan} also scales inversely with the Mach number, β , it follows that in the absence of either source motion ($\beta=0$) or dissipation ($L=0$), the attenuation coefficient diverges to infinity and the evanescent wave vanishes.

Even in the presence of both source motion and dissipation, the attenuation of the evanescent wave is extremely high. Taking values of $\beta=0.147$ and $L=5.9 \times 10^{-8}$ m, as used above to represent a source such as a light aircraft mov-

ing through the atmosphere, we find that $\alpha_{\text{evan}} = 1.1 \times 10^8$ Nepers/m (9.9×10^8 dB/m). Obviously, such heavy attenuation means that the evanescent wave would be almost impossible to detect in most practical circumstances. When the Mach number is very close to unity, the numerator of Eq. (41) falls to near zero and the attenuation coefficient is reduced accordingly, suggesting, on the face of it, that this may be a situation where the evanescent could be detected. However, this is unlikely to be the case, as can be seen in Fig. 4 showing α_{evan} as a function of Mach number; even when the Mach number is as high as $\beta = 0.99$, the attenuation is still in the region of 3.4×10^5 Nepers/m (3×10^6 dB/m). Such high attenuation is a direct result of the low value of L for a typical atmosphere.

In view of its excessively high attenuation, it is almost meaningless to talk of the frequency of the evanescent wave. Nevertheless, as a mere formality, an expression for the (angular) frequency of the evanescent wave is included here, as deduced from Eqs. (27) and (35):

$$\Omega_{\text{evan}} = 2\Omega \frac{\beta^2}{1 - \beta^2}. \quad (42)$$

Although this frequency depends on the Mach number of the source, it is independent of the level of viscous dissipation.

VII. CONCLUDING REMARKS

The properties of the sound field from a moving source in a viscous medium have been investigated theoretically in this article. Based on a simple, one-dimensional model, three types of wave from the moving source are shown to exist in the viscous fluid: an attenuated propagating wave with up-shifted frequency, which is present as the source approaches the receiver; an attenuated propagating wave with down-shifted frequency, which is present as the source recedes from the receiver; and an evanescent wave, which appears at the instant the source passes the receiver and which persists for a very brief time thereafter.

From a theoretical point of view, the evanescent wave is an interesting feature of the received field in that it exists only when source motion and dissipation are both present. When either the Mach number of the source or the viscous dissipation is zero, the attenuation escalates to infinity and the evanescent wave vanishes. Although not zero, dissipation in the atmosphere is sufficiently weak for the evanescent wave to be highly attenuated, so much so that the wave is something of a curiosity and unlikely to be of significance in any practical observation of the sound field from a subsonic source.

A more important effect of the combined presence of source motion and viscous dissipation is the asymmetry introduced between the two propagating waves. Although viscosity has a negligible effect on the Doppler-shifted frequencies, the source motion couples into the attenuation of the two propagating waves in an asymmetrical fashion. Each attenuation coefficient exhibits typical viscous characteristics, scaling as the square of the Doppler-shifted frequency, which is higher on approach than departure. But this is not all. A further asymmetry is introduced into the attenuation coefficients in the form of the factor $(1 \mp \beta)^{-1}$, where β is the Mach number of the source and the upper sign applies on approach, the lower on departure. The combined effect of both forms of asymmetry is an attenuation coefficient for the propagating waves that is higher on approach than departure by the factor $[(1 + \beta)/(1 - \beta)]^3$. This factor increases monotonically with β , taking values in the range 2 to 2000 for Mach numbers between 0.1 and 0.85.

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