Response to "Comments on 'Pore fluid viscosity and the wave properties of saturated granular materials including marine sediments [J. Acoust. Soc. Am. 127, 2095–2098 (2010)]' " (L)

Michael J. Buckingham[a)]
Marine Physical Laboratory, Scripps Institution of Oceanography, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093-0238

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Chotiros and Isakson [J. Acoust. Soc. Am. **127**, 2095–2098 (2010)] raised three main issues concerning the grain-shearing and viscous-grain-shearing (VGS) theories of wave propagation in saturated marine sediments. (1) They introduced the *R*-ratio as a test of the two theories, (2) they then used the *R*-ratio to compare the theories with published measurements of compressional and shear wave properties of laboratory sediments under high confinement pressures, and (3) they pointed out that the VGS theory overestimates the shear attenuation measured during the Sediment Acoustics Experiment 1999 (SAX99) by about an order of magnitude. With regard to the *R*-ratio, it provides an incomplete test of the theories and, moreover, it returns ambiguous results. As for the tests against measurements made under high confinement pressures, they are invalid because the theories are not applicable under such conditions. The third point is correct, but a minor modification to the VGS theory resolves the difficulty.
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I. INTRODUCTION

Chotiros and Isakson[1] provided a critique of Buckingham's grain-shearing[2] (GS) and viscous-grain-shearing[3] (VGS) theories of sound wave and shear wave propagations in saturated granular media. They reached three main conclusions: (1) the dispersion relations for the compressional and shear waves, as predicted by each of the two theories, may be expressed in terms of a single variable, which they designated the *R*-ratio; (2) the GS and VGS theories do not fit the high-pressure data of Prasad and Meissner[4] for either dry or saturated sands; and (3) the VGS model overestimates the shear wave attenuation measured during the ONR-supported Sediment Acoustics Experiment 1999 (Ref. 5) (SAX99) at a frequency of 1 kHz by about an order of magnitude. Each of these three issues is discussed below.

II. THE *R*-RATIO

Chotiros and Isakson's *R*-ratio,[1] their Eq. (13), is based on a reduction of the GS and VGS dispersion relations for compressional and shear waves into a single equation in which all the geophysical parameters appear on one side of the equality sign while all the acoustic parameters appear on the other side. A notable point about the *R*-ratio is that it does not involve the grain-shearing parameters n, γ_p, and γ_s, which characterize the grain-shearing process,[3] but depends only on the wave properties (speeds and attenuations) and the spot frequencies at which they were measured. Chotiros and Isakson's[1] method of analysis follows a similar development to that in Secs. IIB and IIC of Buckingham,[3] and their Eq. (10), which is the central component of the *R*-ratio, is the same as Buckingham's Eq. (16). Chotiros and Isakson[1] asserted that the *R*-ratio makes a comparison of theory with data easy, since the dispersion relations are reduced to a single parameter, whose value is unity in the event of a satisfactory match between theory and data. The *R*-ratio, however, has certain undesirable characteristics, which limit its utility.

For instance, if R were to deviate from unity, representing a mismatch between theory and data, the cause of the departure would be unknown. It could be due to a mismatch of the sound speed, the sound attenuation, the shear speed or the shear attenuation, or any combination of all four, since they are blended together in the expression for R. To identify the source of the departure from $R=1$, it would be necessary to revert to direct comparisons of the original dispersion relations with experimental data. At high frequencies, where the GS and VGS theories are essentially the same, such comparisons between the theories and extensive data sets taken from the open literature have already been performed by Buckingham,[6] and in all the cases examined, the theories show a satisfactory fit to the data.

Another limitation of the *R*-ratio is that, even if it were evaluated over a range of spot frequencies, it would yield no explicit information on the frequency dependencies of the wave speeds and attenuations. Its usefulness as a test of the GS and VGS theories is therefore quite restricted, since the frequency dispersion of the wave speeds and the frequency dependencies of the attenuations are important factors when it comes to assessing the validity of any theory of sound propagation in saturated granular media. Moreover, since the *R*-ratio is just a compressed form of the GS and VGS equa-

[a)]Electronic mail: mbuckingham@ucsd.edu

tions, it is incapable of providing any insights beyond those available from the original dispersion relations themselves. The computational effort required to make direct comparisons between the theories and data is negligible, since the GS and VGS dispersion relations take the form of simple algebraic equations.

A more serious objection to the R-ratio is that it may return a misleading result, because the condition $R=1$ does not correspond to a unique X, Y pair. (X and Y are defined in Refs. 1 and 3.) Thus, it is possible for R to be unity even when the theory does not match the data, since an infinite number of X, Y combinations return a value of $R=1$. To be specific, R takes a value of unity whenever

$$Y = \frac{c_{0b}^2(1+X^2)^2 - c_p^2(1-X^2)^2 + \sqrt{[c_{0b}^2(1+X^2)^2 - c_p^2(1-X^2)^2]^2 + 4c_p^4X^2}}{2c_p^2X}, \quad (1)$$

as may easily be established from Eqs. (11) and (13) in Ref. 1. In effect, the R-ratio may return false positives.

But this is not the only type of ambiguity associated with the R-ratio: It cannot distinguish between the GS theory and the VGS theory when the wave variables in the latter are all measured at the same frequency. Under this condition, the expression for R, as given by Eqs. (11)–(13) in Ref. 1, is exactly the same in the two cases. Even if a genuine (i.e., non-false positive) value of $R=1$ were returned, it would not be possible to tell whether it referred to the GS theory or the VGS theory. A value of $R=1$ is a necessary but not sufficient condition for the theories to match the data.

III. PRASAD AND MEISSNER'S DATA

Prasad and Meissner[4] investigated the effects of high confining pressures on the speeds and attenuations of compressional and shear waves in various samples of oven-dried sands and water-saturated sands. Their measurements were made using a pulse transmission technique in which the center frequency was about 100 kHz, with the samples subjected to confining pressures extending up to approximately 6 MPa (equivalent to 60 atm). Porous pistons were used to compress the samples, allowing the pore fluid to remain unpressurized. Inevitably under such conditions, the grains will have been crushed together, significantly altering the nature of the grain-to-grain contacts. That the grain contacts were indeed modified in the high-pressure experiments is indicated by the fact that, in all the cases reported, the wave speeds and the attenuations at 100 kHz exhibited pronounced dependencies on the confining pressure (see Figs. 2–5 in Ref. 4). Since no dispersion curves were reported by Prasad and Meissner,[4] it is not known whether the compressional and shear wave attenuations in their pressurized sand samples conformed to the near-linear dependence on frequency predicted by the GS theory.

Chotiros and Isakson[1] took Prasad and Meissner's[4] measurements of the compressional and shear wave speeds and attenuations and computed the corresponding R-ratio for each sand sample. They found that in all but one case the R-ratio deviates significantly from unity, and in some cases the R-ratio computed using their Eqs. (11)–(13) turns out to be negative, even though R is defined as the ratio of two real, squared quantities. Chotiros and Isakson[1] concluded that the GS and VGS theories are incompatible with Prasad and Meissner's[4] high-confinement-pressure data.

The GS and VGS theories were developed specifically for water-saturated marine sediments under natural conditions in the sea bed, not for laboratory sand samples under extremely high confinement pressures. The dissipation mechanism that is a central component of the GS and VGS theories is a particular form of inter-granular shearing, which is based on a molecularly thin layer of pore fluid at grain contacts. Under the high confinement pressures of Prasad and Meissner's[4] experiments, the grain contacts may have suffered significant damage and distortion, to the extent that completely new and unknown physical mechanisms governed the wave properties. Since there is no reason to believe that the GS and VGS theories would hold under such extreme conditions, it is inappropriate to use Prasad and Meissner's[4] high-pressure data as a test of the two theories.

IV. SAX99 DATA INCLUDING SHEAR ATTENUATION

Chotiros and Isakson[1] criticized the VGS theory on the grounds that it overestimates the attenuation of the shear wave, as measured during the SAX99 experiments[5] on a medium-sand marine sediment, by about an order of magnitude. The SAX99 data set[5] is unusual, if not unique, in that it contains data on the four wave properties, the compressional and shear wave speeds and both attenuations, along with detailed information on the physical properties of the medium. Using a variety of techniques, the SAX99 compressional wave properties were measured over more than three decades of frequency, from 125 Hz to 400 kHz, resulting in dispersion curves that are extremely well characterized by the experimental data. By comparison, the SAX99 data on the shear wave properties are much more meagre, due to the difficulties of making *in-situ* shear wave measurements: Richardson *et al.*,[5] using bimorph bender elements mounted on the In Situ Sediment geoAcoustic Measurement System (ISSAMS), was able to measure the shear speed and attenuation but at just a single frequency of approximately 1 kHz.

Because of the dearth of data on the shear wave, it received relatively little attention in Buckingham's development of the VGS theory,[3] which, in retrospect, was an oversight. Chotiros and Isakson[1] are correct in noting that the VGS theory yields a shear attenuation that exceeds the value observed in SAX99 by a factor of about 10. A simple resolution of this problem, which should have been incorporated into the original discussion of the VGS theory, is given below.

To take account of the viscosity of the pore fluid, the VGS theory includes the function

$$g(\omega\tau) = \left(1 + \frac{1}{j\omega\tau}\right)^{-1+n}, \quad (2)$$

where the variables are as defined by Buckingham.[3] The viscoelastic time constant in Eq. (2) may be expressed as

$$\tau = \frac{1}{2\pi f_t}, \quad (3)$$

where f_t is a threshold frequency above which the effect of pore fluid viscosity decays to a negligible level. In the original VGS theory,[3] it is implicit that the threshold frequency is the same for the compressional wave and the shear wave, and therein lies the origin of the mismatch to the shear attenuation.

Rather than a threshold frequency, a more plausible criterion is that the onset of pore fluid viscosity is governed by wavelength. Letting λ_t be the threshold wavelength, below which the effect of pore fluid viscosity decays to zero, then the viscoelastic time constants for the compressional and shear waves will be different. For the compressional wave

$$\tau = \tau_p = \frac{\lambda_t}{2\pi c_p} \tag{4a}$$

and for the shear wave

$$\tau = \tau_s = \frac{\lambda_t}{2\pi c_s}, \tag{4b}$$

from which it follows that the viscoelastic time constants for the compressional and shear waves are related as follows:

$$\tau_s = \frac{c_p}{c_s}\tau_p. \tag{5}$$

Thus, τ_s is larger than τ_p by a factor equal to the ratio of the wave speeds, which is of the order of ten.

The VGS dispersion relations may now be expressed in terms of the new viscoelastic time constants. For the compressional wave, the speed and attenuation are

$$c_p(\omega) = \frac{c_0}{\mathrm{Re}[1+\chi(j\omega T)^n g(\omega\tau_p)]^{-1/2}}, \tag{6a}$$

$$\alpha_p(\omega) = -\frac{\omega}{c_0}\mathrm{Im}[1+\chi(j\omega T)^n g(\omega\tau_p)]^{-1/2}, \tag{6b}$$

and for the shear wave

$$c_s(\omega) = \frac{\sqrt{\gamma_s/\rho_0}}{\mathrm{Re}[(j\omega T)^n g(\omega\tau_s)]^{-1/2}}, \tag{7a}$$

$$\alpha_s(\omega) = -\omega\sqrt{\frac{\rho_0}{\gamma_s}}\,\mathrm{Im}[(j\omega T)^n g(\omega\tau_s)]^{-1/2}, \tag{7b}$$

where the notation is the same as that used by Buckingham.[3]

To distinguish the four dispersion relations in Eqs. (6) and (7) from the original VGS theory, they will be referred to as the VGS(λ) version of the theory. It should be noted that the grain-shearing parameters, n, γ_p, and γ_s (and hence χ) in the VGS(λ) theory, are determined using the same matching technique introduced by Buckingham[3] in connection with the VGS theory, and thus, for a given data set such as SAX99, n and χ will be exactly the same in the two versions. If τ_p is taken to be equal to τ, then the compressional wave dispersion curves predicted by the two versions of the theory are identical. Thus, the effects of introducing $\tau_s \neq \tau_p$ are only apparent in the shear wave dispersion curves.

For a comparison of the VGS(λ) theory with the SAX99 experimental measurements, all four expressions in Eqs. (6) and (7) have been evaluated as functions of frequency using the parameter values shown in Table I. The resultant VGS(λ) wave speeds and attenuations are plotted in Fig. 1, with the SAX99 data included for comparison. Since $\tau_p=\tau$, and n and χ have the same values as previously,[3] the VGS(λ) curves in Figs. 1(a) and 1(b) for the compressional wave are identical to their counterparts from the VGS theory (shown as the red curves in Figs. 1a and 1b of Ref. 3). The shear speed and attenuation curves from the two theories differ, however, as illustrated in Figs. 1(c) and 1(d), respectively. The single data point represented by the solid green circle in Fig. 1(d) is Richardson's value of 30 dB/m for the shear attenuation,[5] with the error bars denoting the spread in his measurements, ranging from 21 to 40 dB/m. Clearly, the black VGS line in Fig. 1(d) overestimates the shear attenuation, as noted by Chotiros and Isakson,[1] whereas the red VGS(λ) curve is consistent with Richardson's[5] data point.

TABLE I. Parameter values used in VGS(λ) theory for comparison with SAX99 experimental data.

Material parameter	Symbol	SAX99
Density of pore fluid (kg/m^3)	ρ_w	1023
Bulk modulus of pore fluid (Pa)	K_w	2.395×10^9
Sound speed in pore fluid (m/s)	c_w	1530.1
Density of grains (kg/m^3)	ρ_s	2690
Bulk modulus of grains (Pa)	K_g	3.2×10^{10}
Porosity	N	0.385
Bulk density of sediment (kg/m^3)	ρ_o	2048.2
Bulk modulus of sediment (Pa)	κ_o	5.557×10^9
Wood's speed (m/s)	c_o	1647.1
Material exponent	n	0.0617
Grain-shearing coefficient	χ	0.0724
Compressional coefficient (Pa)	γ_p	3.796×10^8
Shear coefficient (Pa)	γ_s	1.711×10^7
Compressional viscoelastic time constant (ms)	τ_p	0.12
Shear viscoelastic time constant (ms)	τ_s	1.77
Matching parameters		
Sound speed (m/s)/frequency (kHz)		1770.1/38
Sound attenuation (dB/m)/frequency (kHz)		10.12/38
Shear speed (m/s)/frequency (kHz)		120/1

V. CONCLUDING REMARKS

In the final sentence of their Conclusions, Chotiros and Isakson[1] stated that the tight coupling between the compressional and shear wave speeds and attenuations, which is a fundamentally important feature of the GS and VGS theories [and also, of course, the VGS(λ) version], is not valid and that different material exponents (the parameter n in Table I) may be needed for shear and compressional waves. Such a suggestion, that the compressional and shear waves are not correlated, is inconsistent with the large body of evidence that has been accumulating in the open literature for several decades.

Hamilton[7,8] and Richardson[9] both developed regression relations linking the compressional wave speed to the shear wave speed; and Buckingham[6] presented numerous plots of data taken from literature, for instance, compressional speed

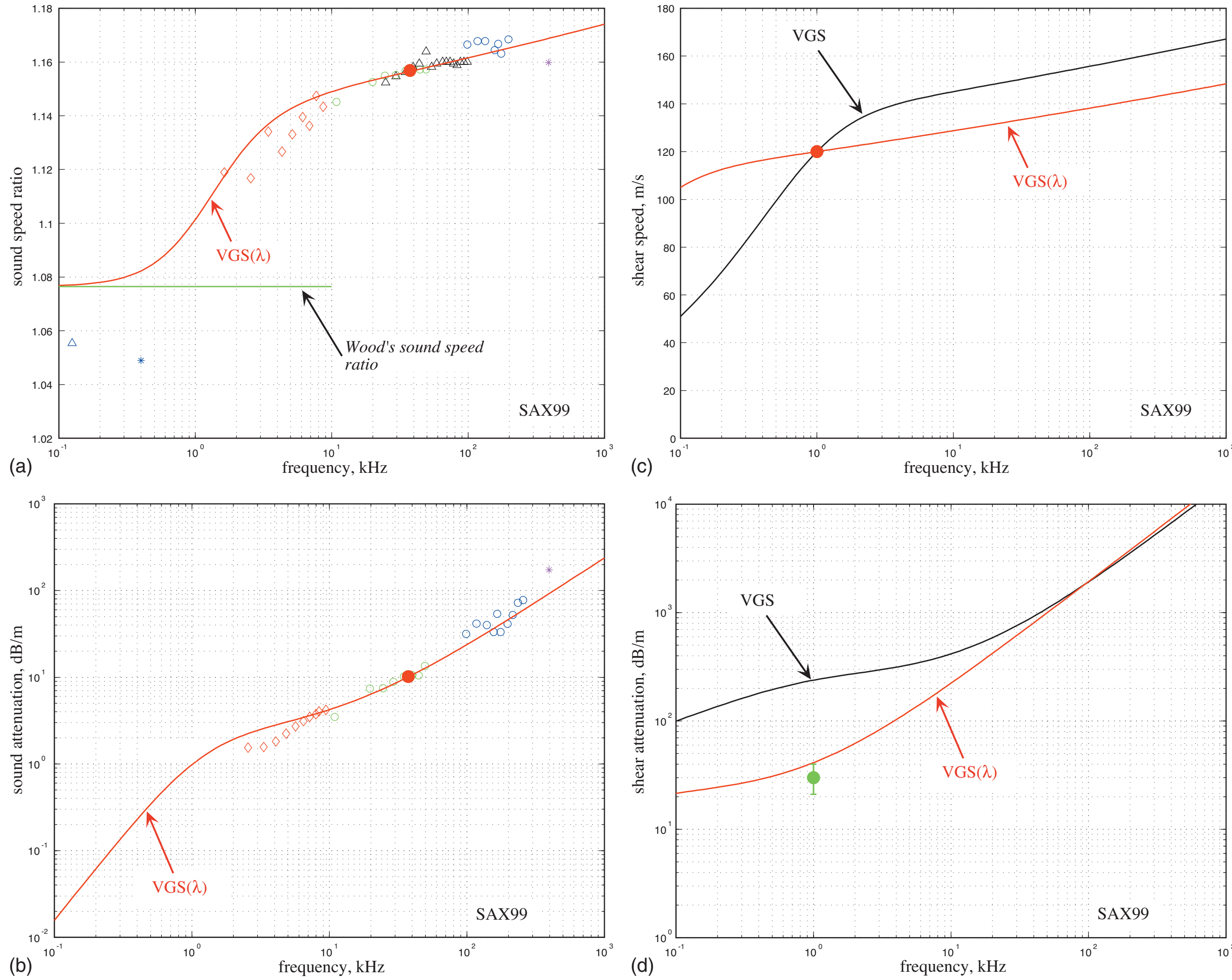

FIG. 1. VGS(λ) theory (red line) evaluated from Eqs. (6) and (7), and VGS theory (black line), for the SAX99 sediment: (a) sound speed ratio, (b) sound attenuation, (c) shear speed, and (d) shear attenuation. The large solid red circles in (a)–(c) are the points at which the theories were matched to the data. The large solid green circle in (d) is the measurement of the shear attenuation.

versus porosity and shear speed versus porosity, from which it may be inferred that the two wave speeds are strongly coupled. It is difficult to understand how Chotiros and Isakson[1] can reconcile their assertion that the compressional and shear waves are not tightly coupled with the overwhelming experimental evidence to the contrary.

Moreover, the idea that different material exponents are required for the compressional and shear waves, as suggested by Chotiros and Isakson,[1] is unnecessary, as exemplified in the analysis of the VGS(λ) version of the viscous-grain-shearing theory developed above. In any case, such arbitrary tampering with the material exponent n is not in the spirit of the GS and VGS theories, which are based as far as possible on physical mechanisms rather than *ad-hoc* data-fitting techniques.

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